Experimental assessment and prediction of temporal scour depth around a spur dike

Abstract

Spur dikes are river training structures that have been extensively used worldwide for towards enhancing flood control and the stability of embankments and riverbanks. However, scour around spur dikes can be a major problem affecting their stability and hydraulic performance. The precise computation of temporal scour depth at spur dikes is very important for the design of economical and safe spur dikes. This study focuses on experimentally assessing the temporal variation of scour depth around a vertical wall spur dike and identifying the parameters, which mostly influence spur dike performance for a channel bed surface comprised of sand-gravel mixtures. In the current study, the authors did physical experiments in a flume based study to obtain new data, aimed at deriving a new predictive model for spur dike scour and comparing its performance to others found in the literature. It was found that the dimensionless temporal scour depth variation increases with an increase in (i) the threshold velocity ratio, (ii) the densimetric Froude number of the bed surface sediment mixture, (iii) the flow shallowness (defined as the ratio of the approach flow depth, y, to the spur dike's transverse length, l), and (iv) the flow depth-particle size ratio. It is also concluded that the temporal scour depth variation in the sediment mixture is influenced by the non-uniformity of sediment and decreases with an increase in the non-uniformity of the sediment mixture. A new mathematical model is derived for the estimation of temporal scour depths in sand-gravel sediment mixtures. The proposed equation has been calibrated and validated with the experimental data, demonstrating a good predictive capacity for the estimation of temporal scour depth evolution.

Keywords: Spur dike, Sand-gravel sediment mixture, Clear-water scour, Time-dependent scour depth.

1. Introduction

Scour and erosion processes at the bed surfaces and banks of natural rivers as well as built waterways (i.e. irrigation or navigation channels), are major challenges that need to be tackled efficiently for proper water resources management. Scour processes in water bodies are the primary cause for geomorphic changes leading to land loss, excessive sediment yield, and subsequent degradation of water quality. Spur dikes are man-made hydraulic structures, commonly used to divert flow from channel banks and protect them against erosion. Generally, spur dikes are comprised of vertical built walls placed at an angle to the mean flow direction. Spur dikes achieve their major function of protecting stream banks (i) directly via diverting the flowing water away from the stream banks, and (ii) indirectly by reducing the flow velocity. Spur dikes are amongst the most hydraulically and cost effective river training structures that can be built at the channels' banks (Zhang et al., 2012), to safeguard their stability, maintain a channel's navigation capacity, as well as help maintain water quality and restore aquatic habitat (Lodhi et al., 2016; Yazdi et al., 2010).

Flow structures around a spur dike may greatly vary, exhibiting three-dimensional features in its vicinity. These complex flow structures scale with the size of the spur dike and flow velocity, containing sufficient energy for the removal of bed material (Diplas et al., 2008; Valyrakis et al., 2010; Valyrakis et al., 2013), leading to the generation of local scour. The study of scour processes around spur dikes is very important for river training and can provide practical guidelines that hydraulic engineers and practitioners may use for their effective operation and management (Zhang et al., 2018).

Local scour is well defined in fluvial hydraulics as the removal of sediment particles from alluvial streams. It is an important geomorphic process that affects virtually all hydraulic infrastructure, ranging from bridge piers (Valyrakis et al., 2015) to embankments and riverbanks (Liu et al., 2017), and is recognized as the primary cause for the failure of spur dikes (Kothyari et al., 2007; Pandey et al., 2017; Zhang et al., 2018). Local scour is classified into two categories; (i) Clear-water scour and (ii) Live-bed scour. For clear-water scour, there is no sediment transport along the stream, other than a local redistribution of sediment around and downstream the scoured structure, and the threshold velocity ratio is always less than 1 ($U/U_c < 1$), where U is the approach mean flow velocity and U_c is the threshold velocity for initiation of sediment entrainment. For live-bed scour conditions, sediment is transported along the stream and the threshold velocity ratio is always greater than 1 ($U/U_c > 1$). Investigators mainly focus on the typically most commonly occurring clear-water scour processes. Figure 1, shows an upstream view of a typical recent clear water spur dike failure, which occurred in the Gaula river near Haldwani, Uttrakhand (India). Such failures due to clear water spur dike scour processes typically occur around the world, and, thus, form the focus of the current study.

Fig. 1. Typical scour around a spur dike in the Gaula river near Haldwani, Uttrakhand, India.

Spur dike scour has been studied more extensively for uniform sediment beds, compared to mixed sediment bed surfaces (Ezzeldin, 2019; Fazli et al*.,* 2008; Koken & Gogus, 2015; Kothyari & Ranga Raju*,* 2001; Kuhnle & Alonso, 2013; Liang et al. 2019; Mostafa et al., 2019; Nagy, 2004; Pandey et al., 2015; Vaghefi et al*.,* 2009; Zhang et al., 2018; amongst others). Even if a uniform bed surface is a good approximation for many cases, most commonly streambeds are non-uniform, comprised of distinct particle sizes ranging from fine sand to coarse gravel. By definition, non-uniform sediment, comprised of distinct individual fractions in the particle size distribution, exhibit a geometric standard deviation $\left(\sigma_g = \sqrt{d_{84}/d_{16}}\right)$ greater than 1.4 (Raikar & Dey, 2005), where σ_g is the geometric standard deviation of particle size distribution, and d_{84} and d_{16} refer to the maximum dimension (size of sieve opening) of grains with cumulative count at 84% and 16% finer, respectively. In a non-uniform sediment mixture (parent bed material), the removal of finer sediment particles ceases with the development of an armor layer, comprised of coarser sediment (Kothyari et al., 2007). The geometric standard deviation for the armor layer is always smaller to that of its parent particle size distribution (Oliveto & Hager, 2002). Once formed, the armor layer represents a water-worked bed surface, which remains stable, not allowing the further removal of finer sediment. The same processes are observed during the dynamic evolution of scour hole geometry, which gradually progresses until the geometry is stable, indicating that equilibrium scour has been reached. Typically equilibrium scour conditions are assessed using a range of parameters including the mean approach flow features (velocity and depth of flow) and the coarser (armor layer) bed particles and parent bed material characteristics (Sui et al., 2010).

To date, a growing number of laboratory and field studies, along with numerical simulations have been done towards estimating the maximum scour depth at equilibrium conditions (Garde et al., 1961, 1963; Gill, 1972; Koken & Gogus, 2015; Kothyari et al., 2007; Kuhnle & Alonso, 2013; Nagy, 2004; Nasrollahi et al., 2008; Pandey et al., 2015; Radan & Vaghefi, 2016; Vaghefi et al., 2009; Yagci et al., 2016; among others). The temporal evolution of scour depth (d_{st}) also is an important factor for economical and realistic designs, as well as for allowing the safe and efficient hydraulic operation of spur dikes, especially during periods of extreme weather. A hydraulic structure is safe if the estimated scour depth is less than the foundation depth, typically considering a certain safety factor (Guo, 2012), otherwise it may be put at risk and should be closed for public safety during extreme hydrologic conditions. Knowledge of the time required for the development of equilibrium scour may also permit assessing the need for repairs. However, despite its importance, relatively few studies have been done on the literature for the prediction of temporal evolution of scour depth in sediment mixtures (Kothyari et al., 2007; Oliveto & Hager, 2002). Kothyari et al. (1992) developed a method to estimate the evolution of temporal scour depth by considering primary vortex structures shed past spur dikes. The time to reach equilibrium scour, is shorter for uniform sediment compared to non-uniform sediment mixtures (Kothyari et al., 2007; Oliveto & Hager, 2002). It has been stated by Kothyari and Ranga Raju (2001) that a pier or an abutment is similar to a vertical wall spur dike, as far as scour processes are concerned. They also stated that the effect of the developing boundary layer is responsible for smaller scour geometries, for the case of spur dikes.

The foregoing studies demonstrate that the focus of the literature over the last two decades has been on assessing spur dike equilibrium scour, for uniform sediment. Given their importance, the current study aims to consider the importance of flow and sediment parameters on assessing the dynamic temporal evolution of clear-water, spur dike scour, for uniform, as well as for non-uniform, sediment bed surfaces. The current study has two main objectives: (i) to experimentally measure and evaluate the effect of different parameters on the temporal evolution of scour depth at a spur dike, and (ii) to develop a predictive model for the temporal evolution of scour depth, for non-uniform sediment mixtures. Herein, the results of 32 time-dependent laboratory scour experiments, totalling 480 datasets, are used to derive and validate a predictive model for the temporal evolution of spur dike scour.

2. Background theory

Parameters influencing the evolution of temporal scour depth around spur dikes are, their geometry, the properties of the sediment at the bed surface, the mean flow parameters, and time (until equilibrium scour depth has been reached). Time-dependent scour depth (d_{st}) around a vertical wall spur dike in non-uniform sediment mixtures can be written as;

$$d_{st} = f\left(d_{50}, \sigma_g, \rho_s U_c, U, \rho, y, l, t\right) \tag{1}$$

In Eq. 1, d_{50}, σ_g, U_c and ρ_s are the sediment parameters; ρ, U and y are the flow parameters; l is the spur dike length scale; and t refers to the time of the dynamically evolving scour process. Specifically, d_{50} is the median diameter of the bed surface material, ρ_s is the density of sediment, ρ is the density of water, y is the flow depth. From Eq. 1, some non-dimensional parameters can be considered, such as, the non-dimensional time function, T, the densimentric Froude number, F_d, and the densimetric Froude number for a sediment mixture, F_{sm} (Oliveto & Hager, 2002). These are expressed as follows:

$$T = t\left\{{\sigma_g}^{1/3}\sqrt{(S-1)gd_{50}}/(l^2 y)^{1/3}\right\} \tag{2}$$

where $S = \rho_s/\rho$ is the specific gravity of the solid grains and g is gravitational acceleration.

$$F_d = U/\sqrt{(S-1)gd_{50}} \tag{3}$$

$$F_{sm} = {\sigma_g}^{-1/3} F_d \tag{4}$$

Oliveto and Hager (2002) derived an equation to calculate the time-dependent scour depth, as given by Eq. 5, based on the geometric standard deviation, playing an important role for non-uniform sediment mixtures, and the densimetric Froude number, affecting the temporal evolution of scour depth. For non-uniform sediment mixtures, they introduced the inception densimetric particle Froude number, F_{di}:

$$\frac{d_{st}}{R_L} = 0.068\sigma_g^{-1/2} F_d^{1.5} \log T \qquad (5)$$

where $R_L = (l^2y)^{1/3}$, is the reference length. Eq. 5 applies for clear-water conditions and when the densimetric Froude number is greater than the densimetric Froude number at the sediment bed inception conditions around the abutment (Hager & Oliveto, 2002).

Kothyari et al. (2007) modified the approach of Oliveto and Hager (2002) on the basis of the difference of F_d-$F_{d\beta}$, where $F_{d\beta}$ is the densimetric Froude number for the inception of scour at spur dikes. They proposed a time dependent scour depth equation, given by Eqs. 6-10:

$$\frac{d_{st}}{R_L} = 0.272\sigma_g^{-1/2}\left(F_d - F_{d\beta}\right)^{2/3} \log\ T \qquad F_d > F_{d\beta} \qquad (6)$$

where $F_{d\beta}$ is expended as:

$$F_{d\beta} = \left[F_{di} - 1.26\Sigma\Sigma_s\Sigma_{ca}\beta^{\Sigma/4}\left(\frac{R_h}{d_{50}}\right)^{1/6}\right]\sigma_g^{1/3} \qquad (7)$$

where F_{di} is computed as:

$$F_{di} = 2.33 D_*^{-0.25}\left(\frac{R_h}{d_{50}}\right)^{1/6} \qquad D_* \leq 10 \qquad (8)$$

$$F_{di} = 1.08 D_*^{1/12}\left(\frac{R_h}{d_{50}}\right)^{1/6} \qquad 10 < D_* < 150 \qquad (9)$$

$$F_{di} = 1.65\left(\frac{R_h}{d_{50}}\right)^{1/6} \qquad D_* \geq 150 \qquad (10)$$

In the foregoing equations (Eqs. 6-8), R_h is the hydraulic radius, F_{di} is densimetric Froude number for incipient sediment entrainment, $D_* = \left({(S-1)g}/{\upsilon^2}\right)^{1/3} d_{50}$ is the dimensionless particle size, and v is the kinematic viscosity of water. For sand-gravel mixtures, d_{50} can be approximated by the median particle size of sediment mixture, $d_{50} = d_a$ with $d_a = \sum p_i d_i$, where p_i is the percentage of sediment by weight corresponding to the size fraction d_i.

Further, Σ is the spur dike shape factor, $\Sigma = 1$ for vertical wall spur dikes; Σ_s is the submergence factor for non-submerged spur dikes (taken as 1), Σ_{ca} is the cascade factor, $= 1.25$ for vertical wall spur dikes; and $\beta = l/B$ is the spur dike obstruction factor, with B being the channel width. Equation 6 has been tested for a range of values of F_d-$F_{d\beta}$. Similar to Oliveto and Hager (2002), it was also concluded that the temporal evolution of the scour depth is greatly influenced by σ_g, for non-uniform mixtures.

3. Experiments

All experiments were done in a flume that has 24.0 m long, 1.0 m wide, and 0.5 m deep. The working test section, has dimensions of 4.0 m × 1.0 m × 0.4 m, starting 12 m from the flume's inlet. All experiments were done under clear water scour conditions. Figure 2 shows a sketch of the flume, with its test section.

For each of the experimental runs, the working section of the flume was fully filled with one of the two pre-prepared sediment mixtures, up to the desired bed surface level, defined by concrete blocks placed upstream and downstream the working section. The first sediment mixture is comprised of 50% sand (d_{50s} = 0.27 mm, σ_g = 1.22) and 50% gravel (d_{50g} = 2.7 mm, σ_g = 1.21), and the second comprises of 50% sand (d_{50s} = 0.27 mm, σ_g = 1.22) and 50% gravel (d_{50g} = 3.1 mm, σ_g=1.18), where d_{50s} is the median diameter of sand and d_{50g} is the median diameter of gravel. A linear profiler has been used to manually level the sediment bed surface in the working section. Four vertical rectangular wall spur dikes, with different transverse length scales (of 6.0, 9.0, 11.5, and 14.0 cm), were used as physical models for the spur dikes. All experiments were done for an unsubmerged spur dike case. The water supply into the flume was regulated by a valve, at the inlet tank upstream of the flume. A sharp crested, pre-calibrated weir was provided at the end of flume to measure the flow rate. Scour depth (d_{st}) was measured with a point gauge at different time intervals, i.e. t = 1, 3, 5, 10, 15, 30 and 60 min up to several hours, until an equilibrium scour geometry was reached (demonstrated by current invariance of any measurable scour depth with the passage of time). Each experiment in the current study was done for 20 hr. However, equilibrium time (t_e) varied in the range of 8-15 hr. For checking the equilibrium scour, the scour depth was measured over time at the nose and junction of the spur dike every 30 min after the equilibrium time with an accuracy of ±1 mm.

Before the start of each experimental run, the spur dike was fixed vertically at the right side of the channel wall (as viewed from upstream), 14.0 m from the flume entrance (Fig. 2). The working-section was perfectly leveled with respect to the longitudinal slope of the flume bed and then covered with a Perspex sheet. The tail gate and inlet valve were used to obtain predetermined flow conditions in the flume. Once the predetermined flow condition was established, the Perspex sheet was removed very carefully so that no scour occurred due to this process. This allowed the time required for the scour depth (d_{st}) to develop to be accurately measured. For each test, hydraulic parameters and scour depth data are listed in Table 1. For vertical wall elements like spur dikes and abutments scour always starts just downstream of the corner region protruding into the upstream flow, than migrates first toward the upstream side (Oliveto & Hager, 2002). This process is done over a very small time interval. In the current study, scour depth was measured at different time intervals at the upstream nose of the spur dike, as this is the expected location at the equilibrium scour condition where the maximum scour depth occurs (Kothyari & Ranga Raju, 2001; Lodhi et al., 2016; Pandey et al., 2015). For the current study, all experiments refer to clear water scour conditions. Table 1 also lists the threshold velocity and shear velocity for the bed surface particles, calculated using the same methodology as in Sui et al. (2010). Figure 3, shows a characteristic three-dimensional visualization for the bed surface elevation, for one of the experimental runs, demonstrative of the scour hole geometry obtained at equilibrium scour conditions.

Fig. 2. Layout top (plan) and side (elevation) views of the flume, showing the working

section.

Fig. 3. Side view of a typical equilibrium scour hole around the spur dike and the corresponding topographical representation of the measured bed surface elevation (see inset at top left).

Table 1. Experimental conditions and equilibrium spur dike scour hole, obtained from the experiments done in the current study.

Run	l (m)	y (m)	U (m/s)	d_{50s} (mm)	d_{50g} (mm)	F_d	U/U_c	t_e (min)	d_{se} (m)
R1	0.140	0.112	0.41	0. 27	2.7	2.66	0.90	480	0.149
R2	0.140	0.105	0.35	0. 27	2.7	2.27	0.77	660	0.111
R3	0.140	0.1	0.31	0. 27	2.7	2.01	0.68	720	0.095
R4	0.140	0.09	0.28	0. 27	2.7	1.82	0.61	840	0.072
R5	0.115	0.112	0.41	0. 27	2.7	2.66	0.90	540	0.128
R6	0.115	0.105	0.35	0. 27	2.7	2.27	0.77	690	0.091
R7	0.115	0.1	0.31	0. 7	2.7	2.01	0.68	780	0.076
R8	0.115	0.09	0.28	0. 27	2.7	1.82	0.61	840	0.057
R9	0.090	0.112	0.41	0. 27	2.7	2.66	0.90	600	0.104
R10	0.090	0.105	0.35	0. 27	2.7	2.27	0.77	660	0.078
R11	0.090	0.1	0.31	0. 27	2.7	2.01	0.68	780	0.063
R12	0.090	0.09	0.28	0. 27	2.7	1.82	0.61	870	0.051
R13	0.060	0.112	0.41	0. 27	2.7	2.66	0.90	630	0.074
R14	0.060	0.105	0.35	0. 27	2.7	2.27	0.77	720	0.058
R15	0.060	0.1	0.31	0. 27	2.7	2.01	0.68	810	0.045
R16	0.060	0.09	0.28	0. 27	2.7	1.82	0.61	900	0.038
R17	0.140	0.112	0.41	0. 27	3.1	2.49	0.84	510	0.127

R18	0.140	0.105	0.35	0. 27	3.1	2.13	0.71	600	0.096
R19	0.140	0.1	0.31	0. 27	3.1	1.89	0.63	660	0.074
R20	0.140	0.09	0.28	0. 27	3.1	1.70	0.57	810	0.057
R21	0.115	0.112	0.41	0. 27	3.1	2.49	0.84	570	0.107
R22	0.115	0.105	0.35	0. 27	3.1	2.13	0.71	660	0.078
R23	0.115	0.1	0.31	0. 27	3.1	1.89	0.63	720	0.059
R24	0.115	0.09	0.28	0. 27	3.1	1.70	0.57	840	0.047
R25	0.090	0.112	0.41	0. 27	3.1	2.49	0.84	600	0.086
R26	0.090	0.105	0.35	0. 27	3.1	2.13	0.71	660	0.068
R27	0.090	0.11	0.31	0. 27	3.1	1.89	0.63	750	0.054
R28	0.090	0.13	0.28	0. 27	3.1	1.70	0.57	870	0.041
R29	0.060	0.12	0.41	0. 27	3.1	2.49	0.84	600	0.058
R30	0.060	0.11	0.35	0. 27	3.1	2.13	0.71	720	0.043
R31	0.060	0.13	0.31	0. 27	3.1	1.89	0.63	780	0.037
R32	0.060	0.12	0.28	0. 27	3.1	1.70	0.57	870	0.032

4. Results

The time dependent scour depths with different sand-gravel mixtures were measured at the nose and the spur dike-wall junction. Figure 4 shows the comparison of time dependent scour depth variation at the nose of the spur dike and spur dike-wall junction. Figure 4a shows that the temporal scour variation at the nose of the spur dike is always higher than at the spur dike-wall junction. From the measurements of the bed surface elevation (Figs. 4a-b), it can be observed that 83% to 89% of the equilibrium scour depth at the spur dike's nose developed within 6% of the time needed to reach equilibrium, while these values range from 75% to 90% for the spur dike-wall junction. Figures 4b-c demonstrate the rate of change of bed surface elevation, and clearly indicate that scour at the initial stage develops rapidly (similar to Figs. 4a-b). However, after 40% of the equilibrium sour time, the rate of change of scour depth almost flattens, being particularly slow for the scour at the nose (compared to 60% of the equilibrium scour time for the spur dike-wall junction location).

It can be said that the scour rate at the nose of the spur-dike is greater than at the upstream wall junction and it takes more time to reach equilibrium, as shown in Figs. 4b-c. The small scour rate at the spur dike wall junction means it more quickly reaches the equilibrium state. For an extreme case, if the flow is low enough and scour only occurs at the nose, the equilibrium time of scour at the wall junction is very low or should be zero. Figure 4d shows the variation between the ratios of temporal scour depths at the nose and the wall junction and scour time, with respect to the equilibrium scour time. It can be clearly observed that scour progresses quickly in its initial stage (< 6% scour time). All tests show similar rates of change of the scour depth, but for 6%-25% of the scour time, the decreasing rate of the scour depths ratio increases with an increase in the approach velocities (R5 to R8). However, at the final stage of the scour (80%-100%), the scour depths ratios increase with increasing approach velocities (R5 to R8). For any particular test case, the scour depth ratio initially decreases rapidly, but after 25% of the equilibrium scour time, the rate of decrease of the scour depth ratio becomes very small or constant, as can be seen in Fig. 4d.

Fig. 4 (a) Temporal variation of scour depth at the nose (–●– R5, –■– R6, –◆– R7, –▲– R8) and junction (–○– R5, –□– R6, –◇– R7, –△– R8) of the spur dikes studied herein, for a range of flow conditions, **(b)** percentage scour variation with time at nose of spur dike, **(c)** percentage scour variation with time at spur dike-wall junction and **(d)** ratio of the scour depth at the nose and the wall junction.

5. Discussions

The foregoing results are used to formulate a discussion on how the temporal evolution of spur dike scour depth may be affected by flow and spur dike design parameters, including the effect of a non-uniform bed surface (comprised of mixed sediment). Then a re-evaluation of recently presented equations is done, and the study concludes by proposing a new predictive model for spur dike scour depth, yielding demonstrably better results than for previous equations.

5.1 *Influence of mean flow parameters and spur dike length on temporal scour depth variation*

Figures 5a-d, show the variation of d_{st} vs. R_L, d_{st}/R_L vs. y/l, d_{st}/R_L vs. log (T) and d_{st}/R_L vs. F_d. The scour depth at the spur dike's nose measured at time t (d_{st}) is found to consistently increase with increasing reference length (R_L), transverse length of the spur dike and approach flow depth, as shown in Fig. 5a. Similar results were obtained by Oliveto and Hager (2002), for vertical wall abutments. Figure 5a shows that the temporally evolving scour depth, increases with increasing U/U_c and reaches maximum for U/U_c = 0.80-0.90.

Figure 5b, shows the influence of the ratio of the approach flow depth to the transverse length of the spur dike (y/l) on the dimensionless temporal scour depth variation. The ratio of y/l is also known as flow shallowness (Lanca et al., 2013). For a particular spur dike length, the temporal scour depth increases with flow shallowness. The effect of the flow shallowness is more noticeable for the case of smallest transverse length of the spur dike. The rate of temporal scour depth variation increases with a decrease in the transverse length of spur dike.

Figure 5c, shows the effect of dimensionless time parameter (T) on d_{st}/R_L with different ranges of y/d_{50}. For different ranges of y/d_{50}, Fig. 5c clearly illustrates that the variation of temporal scour depth increases with an increase in T. Figure 5c, also shows that the temporal scour depth variation increases with the increase in the ratio y/d_{50}. As the scour hole enlarges, mainly with the removal of fine sediment, the developing armor layer slowly raises the effective threshold bed-shear stresses, which reduces the growth of the scour-hole, (Raikar & Dey, 2005). The rate of temporal scour depth variation increases with the y/d_{50} ratio, as can be seen in Fig. 5c. Figure 5c indicates that the temporal scour depth variation becomes approximately constant after log $T = 6.5$.

A number of previous studies, have demonstrated the strong dependence of scouring on mean flow as well as sediment properties (Ezzeldin, 2019; Kothyari & Ranga Raju, 2001; Pandey et al., 2018; Yagci et al., 2016; Zhang et al., 2018). Hence, it is necessary to relate the temporal scour depth evolution to the particle densimetric Froude number, which is a function of the approach mean velocity and particle size, i.e. $d_{st}/R_L \approx f(F_d)\times f(T)$, where $f(F_d)$ is the function of densimetric Froude number and $f(T)$ is the time function. Figure 5d shows the variation of d_{st}/R_L vs F_d.

Figure 5d shows the influence of the densimentric particle Froude number (F_d) on the maximum dimensionless temporal scour depth (d_{st}/R_L) variation. Figure 5d, clearly indicates that the ratio d_{st}/R_L increases with increasing F_d. F_d, mainly depends on the approach velocity and particle size. It has already been explained that the scour depth variation increases with increasing approach flow velocity. Thus, it is observed that the maximum temporal scour depth variation increases with a decrease in the median diameter of the sediment mixture.

Fig. 5. Influence of mean flow parameters and spur dike length on the temporally varying scour depth, at the spur dike's nose (d_{st}): **(a)** d_{st} vs. R_L, **(b)** d_{st}/R_L vs. y/l, **(c)** d_{st}/R_L vs. *log* (T) and **(d)** d_{st}/R_L vs. F_d.

5.2. *Influence of sediment mixture characteristics on the temporal scour depth*

The current study mainly deals with a non-uniform sediment bed and non-uniform sediment is well characterized by the geometric standard deviation of particle size distribution. It has been observed by the authors that the temporal scour depth variation for sediment mixtures is influenced by the non-uniformity of sediment. Hence, the effect of sediment mixture on d_{st} was analyzed in terms of the sediment mixture Froude number $\left(\mathrm{F}_{sm} = \sigma^{-1/3}\mathrm{F}_d\right)$. Figures 6a-d show the effect of F_{sm} on temporal scour depth evolution at different times (t = 1, 15, 60, and 120 min).

Sediment's non-uniformity has a significant effect on temporal scour depth evolution. For a fixed time and transverse length of the spur dike, Figs. 6a-d clearly show that the maximum temporal scour depth increases with an increase of the sediment mixture's Froude number, which indicates that the temporal scour depth increases with decreasing sediment non-uniformity. It has also been observed, development of an armor layer in the scoured region begins causing in an exposure of coarser gravel sizes due to washing out of the finer gravel particles. The scour rate at its initial stage (i.e. before 60 min) is very high. Up to 80% of the scour occurs before this time, as can be seen in Figs. 6a-d. The scour rate with time was very slow after this initial stage, because of the formation of a protective armor layer. Once a stable armor layer forms around the spur dike, no further removal of sediment occurs in the scour hole.

Fig. 6. Influence of sediment mixture in terms of F_{sm} on temporal scour depth variation.

5.3. *A re-assessment of the performance of past models*

Measured values of the scour depth (d_{st}) at predefined time intervals (t), at the location of maximum scour depth at the upstream nose of the spur dike, are shown in Figure 7, for different transverse lengths of the spur dike and flow conditions. In the current study, the time dependent scour depth has been also calculated using the equations given by Oliveto and Hager (2002) and Kothyari et al. (2007), as shown in Figs. 7a-h. These estimated values of time dependent scour depths also are compared with observed values from the experiments presented in the current study. It is observed that the model predictions, using either of the previous models, are in a good agreement with the obtained experimental data, for the larger transverse lengths of the examined spur dikes (e.g., l= 11.5 and 14.0 cm – see Figs. 7a-b). For the shorter spur dike transverse lengths, both equations exhibit an increasing difference between observed and computed values of time-dependent scour depths (Figs. 7c-d, g-h).

Fig. 7. Comparison of the obtained experimental data of the temporal scour depth variation with the predictions calculated using the equations of Oliveto and Hager (2002) and Kothyari et al. (2007) for a range of spur dike lengths and flow rates.

5.4. *A new model for temporal scour depth*

Theoretically, the temporal scour depth around a vertical wall spur dike has been briefly explained in the foregoing sections. Equations 1-4 show the dependent parameters of the temporal scour depth evolution relations. In a sand-gravel sediment mixture, the characteristics of the parent bed material and armor-layer material are the most important to consider (Sui et al., 2010). Using Buckingham's pi theorem, Eq. 1 may be written in dimensionless form as:

$$\frac{d_{st}}{R_L} = f\left(F_{sm}, \frac{l}{d_{50}}, T\right) \tag{11}$$

It has been stated by previous researchers that the element Reynolds number plays a minor role for scour processes (Lim & Chiew, 2001; Sui et al., 2010). The armor-layer development processes and thickness are typically not explicitly considered as a function of temporal scour depth variation. The importance of the different parameters for the temporal scour depth variation, included in Eq. 11, has been demonstrated in the foregoing sections. Using these parameters, an empirical relation has been derived to calculate the temporal scour depth variation, as given by Eq. 12, employing a nonlinear regression method:

$$\frac{d_{st}}{R_L} = 0.02(F_{sm})^{1.68}\left(\frac{l}{d_{50}}\right)^{0.28}(\log T) \tag{12}$$

Equation 12 offers estimates of the temporal evolution of scour depth around the nose of a vertical wall spur dike under clear-water scour conditions.

5.4.1. *Assessment of the model's performance*

The proposed approach Eq. 12 also is compared with the approach of Oliveto and Hager (2002) and Kothyari et al. (2007) and the results for each of the approaches are assessed in terms of fits of the observed and estimated scour depths to the line of perfect agreement, as shown in Fig. 8.

Figure 9, shows the variation between the total number of percentage data and percentage error. For Oliveto and Hager (2002) and Kothyari et al. (2007), approximately 55% of the datasets were found to be within ± 25% error, whereas all datasets for the proposed relation were found to be within ± 20% error lines. Both previous studies proposed their approaches after analyzing the non-uniform sediment data, but they considered densimetric particle Froude number rather than densimetric mixture Froude number.

Fig. 8. Calculated versus observed values of temporal scour depths at the nose of spur dike using: (a) the proposed approach; (b) Oliveto and Hager's (2002) equation and using Kothyari et al.'s (2007) equation.

Fig. 9. Variation of total number of percentage data versus percentage error.

5.4.2. *Sensitivity analysis*

A sensitivity analysis is undertaken to classify the most critical non-dimensional input parameter of the proposed relation and to check the dependency of different parameters on d_{st}/R_L, as listed in Table 2. The sensitivity analysis is done by using the average values of all non-dimensional values of independent and dependent parameters from the experimental data. An assumption has been made for this analysis, i.e. the errors in each input variable are independent. The average values of non-dimensional input parameters F_{sm}, l/d_{50}, and log (T) for the datasets used in this analysis are 1.39, 64.75, and 6.5, respectively.

If a percentage error ΔΨ in the output is known as the difference between values of output computed for inputs Ω and Ω+ΔΩ, then the percentage error might be estimated as the absolute sensitivity (€=ΔΨ/ΔΩ). Here the output is $\Psi = d_{st}/R_L$ and input $\Omega = F_{sm}$, l/d_{50}, and T. The error also can be expressed in a relative form £ = ΔΨ/Ψ. The error ΔΨ in output is fundamentally the deviation sensitivity with ΔΩ being the error. The relative sensitivity can be expressed ¥ = (ΩΔΨ)/(ΨΔΩ) (Ahmad, 2013).

The sensitivity analysis is completed by changing the each input parameter by ±10%. The outcomes of sensitivity analysis are listed in Table 2, which indicates that F_{sm} is the most sensitive parameter followed by log T, and l/d_{50}. The relative sensitivity (¥) of F_{sm} is nearly 1.7 and 6.8 for log T and l/d_{50}, respectively; for a +10% increase in Ω. However, for a 10% decrease in Ω, the relative sensitivity (¥) of F_{sm} is nearly 1.7 and 5.8 for log T and l/d_{50}, respectively. The computation accuracy of the proposed equation greatly depends on F_{sm}, followed by log T, and l/d_{50}. Thus, the temporal variation of scour depth at the nose of the spur dike, greatly depends on the densimetric mixture Froude number. The detailed results of the sensitivity analysis are listed in Table 2.

Table 2. Outcome of the sensitivity analysis

Percentage change	**Ω**	**ΔΩ**	**ΔΨ**	**€**	**£**	**¥**

$\Delta\Omega$ is +10%	F_{sm}	0.139	0.129	0.928	0.176	1.765
increase in	log (T)	0.649	0.073	0.112	0.100	0.999
Ω	l/d_{50}	6.475	0.019	0.003	0.026	0.260
$\Delta\Omega$ is -10%	F_{sm}	0.139	0.129	-0.871	-0.166	-1.655
decrease in	log (T)	0.649	0.073	-0.109	-0.097	-0.971
Ω	l/d_{50}	6.475	0.019	-0.003	-0.029	-0.287

5.4.3. *Comparative statistical analysis*

In order to check the accuracy of the proposed relation, it has been compared the experimental values with existing relations. The proposed Eq. 12 and previously proposed equations (Oliveto & Hager, 2002; Kothyari et al., 2007) have been evaluated for their accuracy using 480 flume experimental datasets. For an estimation of the difference between calculated and experimental non-dimensional temporal scour depth (d_{st}/R_L), the discrepancy ratio (DR, described by White et al., 1973) is used as an error degree and is described as:

$$\text{DR} = log\left(\frac{d_{st\_computed}}{d_{st\_observed}}\right) \qquad (6)$$

For DR = 0, the computed (d_{st}/R_L) is identical to the experimental (d_{st}/R_L). For negative (positive) values of discrepancy ratio, the computed value of the temporal scour depth evolution is smaller (larger) than the experimental value. Accuracy is described as the frequency of cases for which the DR is within a suitable range for the total number of data, as shown in Fig. 10. It cab be noted from Fig. 10, that the data frequencies within DR = ±0.01 are 104, 116, and 198 for Oliveto and Hager (2002), Kothyari et al. (2007), and the proposed equation, respectively. It can be said that the proposed equation is more accurate to calculate the temporal depths of scour compared to existing relations.

Fig. 10. Variation of experimental data frequency and DR for selected equations.

The performance of the proposed Eq. 12 and previously proposed (Oliveto & Hager, 2002; Kothyari et al., 2007) equations has been also checked using the several statistical indices. These indices are known as way to evaluate the degree of the agreement between computed and experimental values of d_{st}/R_L. If X is the experimental value of d_{st}/R_L and X' is the corresponding calculated value of d_{st}/R_L. Ahmad (2013) defined these statistical indices as:

Coefficient of correlation, $CC = \frac{n\sum XX' - \sum X \sum X'}{\sqrt{n\sum X^2 - (\sum X)^2}\sqrt{n\sum X'^2 - (\sum X')^2}}$ (13)

Mean absolute error, $MAE = \frac{1}{n}\sum_{i=1}^{n}\left|X_i - X_i^i\right|$ (14)

Mean square error, $MSE = \frac{1}{n}\sum_{i=1}^{n}(X_i - X_i')^2$ (15)

Root mean square error, $RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_i - X_i')^2}{n}}$ (16)

Mean absolute percentage error, $MAPE = \frac{100}{n}\sum_{i=1}^{n}\frac{|X_i - X_i'|}{|X_i|}$ (17)

In aforementioned Eqs. 13-17, *n* is the total number of experimental data sets (*n* = 480 in this study). The values of *CC*, *MAE*, *MSE*, *RMSE*, and *MAPE* for each equation are listed in Table 3. The value of *CC* of the proposed equation is higher than for the others, and the values of *MAE*, *MSE*, *RMSE*, and *MAPE* are lower as compared to existing relations. These results mean the proposed quation statistically performs better than the others. However, the proposed equation has an edge over the relations proposed by Oliveto and Hager (2002) and Kothyari et al. (2007). It is apparent that the better representation of sediment non-uniformity by inclusion of the densimetric mixture Froude number, results in better estimates and a higher predictive ability for the proposed equation (Eq. 12).

Table 3. Outcome of the statistical analysis

Relations	CC	MAE	MSE	RMSE	MAPE
Proposed Equation (Eq. 12)	0.98	0.029	0.001	0.037	0.011

Oliveto and Hager (2002)	0.81	0.124	0.024	0.154	0.042
Kothyarie et al. (2007)	0.78	0.159	0.051	0.224	0.054

6. Conclusions

The temporal variation of scour depth around a vertical wall spur dike has been studied experimentally at the upstream nose of the spur dike in a sand-gravel mixture. The influence of different parameters on the temporal scour depth evolution has been discussed in the current study. It is observed that the dimensionless temporal scour depth variation increases with increase in U/U_c, F_d, y/l, y/d_{50}, and T and decreases with an increase in particle size.

A comparison of the measured rates of temporal evolution of scour depth at the spur dike's nose and its junction with the wall also is undertaken. It is found that the temporal scour depth variation remains always less compared to the spur dike's nose (around 8-30% less).

Furthermore, the temporal scour depth variation in sediment mixtures is influenced by the non-uniformity of sediment and increases with increases in densimetric Froude number of the sediment mixture. The temporal scour depth increases with decreases in non-uniformity of sediment.

A new temporal scour depth equation for estimation of maximum scour depth around spur dikes in sand-gravel mixtures, which takes into account the non-uniformity of sediment, has been proposed. The proposed model is derived from dimensional theory using a range of non-dimensional parameters F_{sm}, l/d_{50}, and T, empirically found to affect the non-dimensional temporal scour depth evolution at the nose of the spur dike. The performance of this model also is compared with the approaches of Oliveto and Hager (2002) and Kothyari et al. (2007).

It is found that the proposed model, in addition to having wider applicability to non-uniform bed surfaces, also better matches the experimentally observed values of temporal scour depths, largely due to the consideration of the densimetric Froude number. The performance of the proposed model is found to be superior to the existing temporal scour depth equations, particularly for sand-gravel mixtures, as their comparison to the flume experiments reveals. Statistically, the proposed equation shows good agreement between experimental and computed values, and a better predictive capacity compared to existing models.

List of notation

B	Channel width
d_{50}	Median diameter of sediment mixture
d_{50g}	Median diameter of gravel
d_{50s}	Median diameter of sand
d_{16}	Particle size at 16% finer
d_{84}	Particle size at 84% finer
$d_a = \sum p_i d_i$	Median particle size of sediment mixture
d_i	Particle size corresponds to p_i.
d_{st}	Scour depth at time t
$D_* = \left({(s-1)g}/{\nu^2} \right)^{1/3} d_{50}$	Non-dimensional particle size

$F_d = \frac{U}{\sqrt{(S-1)gd_{50}}}$	Densimetric Froude number
$F_{di} = 2.33D_*^{-0.25}\left(\frac{R_h}{d_{50}}\right)^{1/6}$	Inception densimetric particle Froude number
$F_{d\beta} = \left[F_{di} - 1.26\Sigma\Sigma_s\Sigma_{ca}\beta^{\Sigma/4}\left(\frac{R_h}{d_{50}}\right)^{1/6}\right]\sigma_g^{1/3}$	Densimetric Froude number for inception of scour
$F_{sm} = \sigma_g^{-1/3}F_d$	Froude number of sediment mixture
g	Gravitational acceleration
l	Transverse length of spur dike
n	Total number of datasets
N	Spur dike shape factor (Oliveto & Hager, 2002)
p_i	Percentage of sediment by weight
Q	Flow rate
R_h	Hydraulic radius
$R_L=(l^2y)^{1/3}$	Reference length
S	Specific gravity
t	Time
t_e	Equilibrium time of scour
T	Non-dimensional time function
y	Approach flow depth
U	Approach mean velocity
U_c	Threshold velocity for particle entrainment

ρ	Density of water
ρ_s	Density of sediment
ν	Kinematic viscosity of water
σ_g	Geometric standard deviation of particle sizes
Σ	Spur dike shape factor (Kothyari et al., 2007)
Σ_{ca}	Spur dike cascade factor
Σ_s	Spur dike submergence factor
$\beta = l/B$	Spur dike obstruction factor

References

Ahmad, Z. (2013). Prediction of longitudinal dispersion coefficient using laboratory and field data: relationship comparisons. *Hydrology Research*, *44*(2), 362-376.

Diplas, P., Dancey, C. L., Celik, A. O., Valyrakis, M., Greer, K., & Akar, T. (2008). The role of impulse on the initiation of particle movement under turbulent flow conditions. *Science*, *322*(5902), 717-720.

Ezzeldin, R. M. (2019). Numerical and experimental investigation for the effect of permeability of spur dikes on local scour. *Journal of Hydroinformatics*, 21(2), 335-342.

Fazli, M., Ghodsian, M., & Neyshabouri, S. A. A. S. (2008). Scour and flow field around a spur dike in a 90 bend. *International Journal of Sediment Research*, *23*(1), 56-68.

Garde, R. J., Subramanya, K. S., & Nambudripad, K. D. (1961). Study of scour around spur-dikes. *Journal of the Hydraulics Division*, *ASCE, 87*(6), 23-37.

Garde, R. J., Subramanya, K., & Nambudripad, K. D. (1963). Closure of 'Study of scour around spur-dikes'. *Journal of the Hydraulics Division*, *ASCE, 88*, 167-175.

Gill, M. A. (1972). Erosion of sand beds around spur dikes. *Journal of the Hydraulics Division, ASCE,* 98(9), 1587-1602.

Guo, J. (2012). Pier scour in clear water for sediment mixtures. *Journal of Hydraulic Research*, *50*(1), 18-27.

Hager, W. H., & Oliveto, G. (2002). Shields' entrainment criterion in bridge hydraulics. *Journal of Hydraulic Engineering, 128*(5), 538-542.

Koken, M., & Gogus, M. (2015). Effect of spur dike length on the horseshoe vortex system and the bed shear stress distribution. *Journal of Hydraulic Research*, *53*(2), 196-206.

Kothyari, U. C., Garde, R. C. J., & Ranga Raju, K. G. (1992). Temporal variation of scour around circular bridge piers. *Journal of Hydraulic Engineering*, *118*(8), 1091-1106.

Kothyari, U. C., Hager, W. H., & Oliveto, G. (2007). Generalized approach for clear-water scour at bridge foundation elements. *Journal of Hydraulic*

Engineering, *133*(11), 1229-1240.

Kothyari, U. C., & Ranga Raju, K. G. (2001). Scour around spur dikes and bridge abutments. *Journal of Hydraulic Research*, *39*(4), 367-374.

Kuhnle, R., and Alonso, C. (2013). Flow near a model spur dike with a fixed scoured bed. *International Journal of Sediment Research*, *28*(3), 349-357.

Lança, R. M., Fael, C. S., Maia, R. J., Pêgo, J. P., & Cardoso, A. H. (2013). Clear-water scour at comparatively large cylindrical piers. *Journal of Hydraulic Engineering, ASCE, 139*(11), 1117-1125.

Liang, C., Abbasi, S., Pourshahbaz, H., Taghvaei, P., & Tfwala, S. (2019). Investigation of flow, erosion, and sedimentation pattern around varied groynes under different hydraulic and geometric conditions: a numerical study. *Water*, *11*(2), 235.

Lim, F. H., & Chiew, Y. M. (2001). Parametric study of riprap failure around bridge piers. *Journal of Hydraulic Research*, *39*(1), 61-72.

Liu, D., Valyrakis, M., & Williams, R. (2017). Flow hydrodynamics across open channel flows with riparian zones: implications for riverbank stability. *Water*, *9(9),* 720.

Lodhi, A. S., Jain, R. K., & Sharma, P. K. (2016). Influence of cohesion on scour around submerged dike founded in clay–sand–gravel mixtures. *ISH Journal of Hydraulic Engineering*, *22*(1), 70-87.

Mostafa, M. M., Ahmed, H. S., Ahmed, A. A., Abdel-Raheem, G. A., & Ali, N. A.

(2019). Experimental study of flow characteristics around floodplain single groyne. *Journal of Hydro-environment Research*, *22*, 1-13.

Nagy, H. M. (2004). Maximum depth of local scour near emerged vertical wall spur dike. *Alexandria Engineering Journal*, *43*(6), 819-829.

Nasrollahi, A., Ghodsian, M., & Neyshabouri, S. A. A. S. (2008). Local scour at permeable spur dikes. *Journal of Applied Sciences*, *8*(19), 3398-3406.

Oliveto, G., & Hager, W. H. (2002). Temporal evolution of clear-water pier and abutment scour. *Journal of Hydraulic Engineering*, *128*(9), 811-820.

Pandey, M., Ahmad, Z., & Sharma, P. K. (2015). Estimation of maximum scour depth near a spur dike. *Canadian Journal of Civil Engineering*, *43*(3), 270-278.

Pandey, M., Ahmad, Z., & Sharma, P. K. (2017). Scour around impermeable spur dikes: a review. *ISH Journal of Hydraulic Engineering*, *24*(1), 25-44.

Pandey, M., Sharma, P. K., Ahmad, Z., & Karna, N. (2018). Maximum scour depth around bridge pier in gravel bed streams. *Natural Hazards*, *91*(2), 819-836.

Radan, P., & Vaghefi, M. (2016). Flow and scour pattern around submerged and non-submerged T-shaped spur dikes in a 90° bend using the SSIIM model. *International journal of river basin management*, *14*(2), 219-232.

Raikar, R. V., & Dey, S. (2005). Clear-water scour at bridge piers in fine and medium gravel beds. *Canadian Journal of Civil Engineering*, *32*(4), 775-781.

Sui, J., Afzalimehr, H., Samani, A. K., & Maherani, M. (2010). Clear-water scour around semi-elliptical abutments with armored beds. *International Journal of Sediment Research*, *25*(3), 233-245.

Vaghefi, M., Ghodsian, M., & Salehi Neyshaboori, S. A. A. (2009). Experimental study on the effect of a T-shaped spur dike length on scour in a 90 channel bend. *Arabian Journal for Science and Engineering*, *34*(2), 337.

Valyrakis, M., Diplas, P., & Dancey, C. L. (2013). Entrainment of coarse particles in turbulent flows: An energy approach. *Journal of Geophysical Research: Earth Surface*, *118*(1), 42-53.

Valyrakis, M., Diplas, P., Dancey, C. L., Greer, K., & Celik, A. O. (2010). Role of instantaneous force magnitude and duration on particle entrainment. *Journal of Geophysical Research: Earth Surface*, *115*(F2).

Valyrakis M., Michalis, P., & Zhang, H. (2015), A new system for bridge scour monitoring and prediction, In *Proceedings, 36th IAHR World Congress* (Vol. 28), June 28 –July 03, 2015, The Hague, The Netherlands.

White, W. R., Milli, H., & Crabbe, A. D. (1973). *Sediment transport: an appraisal of available methods.* Hydraulics Research Station Report No. INT 119, Wallingford, U.K.

Yagci, O., Celik, M. F., Kitsikoudis, V., Kirca, V. O., Hodoglu, C., Valyrakis, M., &

Kaya, S. (2016). Scour patterns around isolated vegetation elements. *Advances in Water Resources*, *97*, 251-265.

Yazdi, J., Sarkardeh, H., Azamathulla, H. M., & Ghani, A. A. (2010). 3D simulation of flow around a single spur dike with free-surface flow. *International Journal of River Basin Management*, *8*(1), 55-62.

Zhang, H., Nakagawa, H., & Mizutani, H. (2012). Bed morphology and grain size characteristics around a spur dyke. *International Journal of Sediment Research*, *27*(2), 141-157.

Zhang, L., Wang, H., Zhang, X., Wang, B., & Chen, J. (2018). The 3-D morphology evolution of spur dike scour under clear-water scour conditions. *Water*, *10*(11), 1583.